Geoid determination with Hotine's integral based on terrestrial gravity data in Semarang city

by L.m. Sabri

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Abstract. GNSS surveys can accurately determine the geodetic position, but the elevation values do not have physical meaning for the purposes of engineering work. Physical height can be determined through the gravimetric geoid model representing the relationship between the geometric and physical shapes of the earth. Theoretically, the gravimetric geoid model is a solution of Boundary Value Problems (BVP) to obtain the boundary value in the form of the geoid surface. Before the era of GNSS, a BVP was calculated using Stokes or Molodensky approaches. Another approach that was quite difficult to be practiced in that era was Hotine approach. This approach requires gravity disturbance data on surface of the earth or on the geoid itself. This paper presents an accurate gravimetric geoid modelling based on terrestrial gravity data in Semarang using Hotine's approach. The main gravity data was measured in March 2016 using Scintrex CG-5 gravimeter which has an accuracy of 5 microgal. It covered an area about 1 arc degree squares. Additional gravity data for improving the accuracy of the geoid models was measured by some government and private agencies using analogue gravimeters. It covered Jawa island. The global geopotential model was GECO with maximum degree 2190. The results showed that the computation of Hotine's integral for modelling the geoid of Semarang city had an accuracy of ± 0.044 m. In the region of land subsidence, the geoid model is quite prospective to be implemented as the Vertical Reference System (VRS). Measurement of elevation of a distant point using geoid can be completed in a matter of hours, while using spirit levelling completed within a matter of days.

1. Introduction

In geodesy, accurate elevation can be measured by two methods, namely spirit leveling and geodetic leveling. Measurement of elevation with spirit leveling to produce physical height value is affected by the gravity field equipotential of reference and measurement points [1]. Determining height with a spirit leveling to a certain point located far away from the reference point requires a series of measurements of height difference which is lengthy and time consuming. Measurement a section of spirit leveling within 2 km requires about 1 day of measurement.

Another technique which is accurate and productive for height determination is the measurement using the Global Navigation Satellite System or GNSS. GNSS is the term of satellite-based positioning system that combines satellite of the United States called the Global Positioning System or GPS satellite system, GLONASS belongs to Russia, the satellite system Galileo belongs to the EU, the satellite system COMPASS of China, and other new satellite systems. GNSS survey does not require visibility among points as required in the survey with sprit level. Settings of equipment in GNSS were accomplished only on point of interest or Bench Mark (BM). Those conditions significantly minimize equipment setting errors, so that the remaining error in the survey is just the intrinsic bias of signals from GNSS satellites to receivers [2]. Although GNSS survey can accurately determine the geodetic position, but the resulting height values do not yet have a physical sense as desired in engineering work. Physical height can be calculated through a reduction of the geoid undulation that represents the relationship between the geometric shape of the earth and the physical shape of the earth.

Gravimetric geoid modeling is basically solving Boundary Value Problems or BVP to obtain the boundary value in the form of the geoid surface. Before GNSS era, BVP was solved by Stokes approach and the approach Molodensky [3]. Stokes approach, as used in methods SHGeo, requires gravity data, and the rock density below measured locations [4]. Accuracy of Mexico geoid using SHGeo or Stokes-Helmert Geoid was ± 0.599 m [5], while the accuracy of geoid in Sudan and Tanzania modeled by KTH method were ± 0.290 m [6] and ± 0.278 m [7].

One approach that is quite difficult to be applied before GNSS era is a GPS BVP due to lack of geodetic height data referred to ellipsoid [1]. Recently, coordinate of gravity stations can be measured accurately using relative GNSS positioning. Hotine approach applied to solve GPS BVP requires gravity disturbance data on earth's surface. Testing in Western Australia showed that the geoid modeled from the 60 gravity disturbance data had a similar shape to that modeled geoid from Free-Air gravity anomaly data [8]. The standard deviation Western Australia geoid computed by Stokes approach was ± 0.165 m, while the standard deviation of Hotine's approach was of ± 0.267 m. Application of Hotine's approach for geoid modeling in China using GNSS data and gravity measured in 4870 locations and tested at 65 points introduced an accuracy of ± 0.024 m [9].

In the region of land subsidence, a geoid model with an accuracy of ± 0.024 m is quite prospective to be used as the vertical reference system or VRS. Generally, VRS can be divided in leveling based VRS and geoid based VRS. Elevation measurement of long baseline occupying GNSS receivers and geoid can be completed in a matter of hours, while sprit level measurement of a distant section needs many days of measurement.

Another advantage of the geoid based VRS is a realization that is simpler than the leveling based VRS. Geoid based VRS does not require many physical BM. In a modern measurement system, the role of BM can be replaced by one or several Continuously Operating Reference Station or CORS that can be used as a measurement reference. Deformation of CORS points in the active region can be monitored, because these stations record data continuously. GNSS data can be calculated relative to the International GNSS Service (IGS) stations. Users can utilize the data from the CORS for geodetic position measurement any time with a static or kinematic method, in real time or post processing. Precise orthometric precision depends on quality of GNSS measurement and quality of geoid as reduction model. This research aims to produce precise geoid based on terrestrial gravity data measured in small area.

2. Geoid determination using Hotine's integral

Ellipsoid is a mathematical model to fit the shape of the geoid. Theoretically, gravity potential of geoid is equal to normal gravity potential on ellipsoid. Irregular shapes of the earth and density variation introduce discrepancies between the quantities in normal earth model and actual earth. The difference between the actual gravity potential on the earth's surface (W) and the potential of normal gravity (U) at the earth's surface is called the potential disturbance or potential disruption (T) defined by equation (1) as follow

$$T = W - U \ . \tag{1}$$

The difference between the actual gravity on the earth's surface (g_P) and normal gravity on earth's surface (γ_P) is called the gravity disturbance or disruption of gravity (δg) defined by equation (2) as follow

$$\delta g = g_P - \gamma_P \ . \tag{2}$$

The difference between the actual gravity on the geoid (g_Q) and normal gravity on the ellipsoid (γ) called the gravity anomaly or gravity anomalies (Δg) defined by equation (3) as follow

$$\Delta g = g_Q - \gamma \quad . \tag{3}$$

Measured gravity data are affected by three kind of wave, namely short, medium, and long wave. Short wave was contributed by topographical effect, while long wave was contributed by global gravity of the earth. Medium wave itself is gravity data contributed by density contrast below earth surface. Process of elimination and preservation of short and long wave is known as the Remove-Compute-Restore or RCR. First step of RCR procedure is to calculate the residual gravity disturbance (δg_{res}) by removing the effects of global gravity disturbance (δg_1) from actual gravity observations (δg_{obs}) using equation (4) as follow

$$\delta g_{res} = \delta g_{obs} - \delta g_1 \tag{4}$$

Global gravity disturbance (δg_1) computation is based on degree (n) and order (m) of spherical harmonic coefficient, cosines coefficient ($\Delta \bar{C}_{n.m}$), sinus coefficient ($\Delta \bar{S}_{n.m}$) as follow

$$\delta g_1(\theta, \lambda) = \frac{GM}{R^2} \sum_{n=2}^{N} (n+1) \sum_{m=0}^{n} P_{n.m}(\cos\varphi)\left[\Delta \bar{C}_{n.m}.\cos m\lambda + \Delta \bar{S}_{n.m}.\sin m\lambda\right] . \tag{5}$$

In equation (5), G denotes Newton gravitational constant, M denotes earth's mass, R denotes earth's radius, $P_{n,m}$ denotes legendre function, while φ and λ denote latitude and longitude of point of interest.

Residual gravity disturbance ($\delta g(\varphi', \lambda')$) are then interpolated using krigging method to ensure dense distribution of contribution points. Spherical distance (ψ) between computation point in geodetic coordinate (φ , λ) and contribution point in geodetic coordinate (φ' , λ') can be calculated using equation (6) as follow

$$\cos\psi = \sin\varphi.\sin\varphi' + \cos\varphi.\cos\varphi'.\cos(\lambda' - \lambda) \tag{6}$$

Residual geoid undulation (N_{res}) can be calculated using Brun's formula as seen in equation (7)

$$N_{res}(\varphi, \lambda) = \frac{R}{4.\pi.\gamma} \int_{\lambda'=0}^{2\pi} \int_{\varphi'=-\pi/2}^{\pi/2} \delta g(\varphi', \lambda').H(\psi).\cos\varphi'\, d\varphi'\, d\lambda' \tag{7}$$

where Hotine function (H) is declared in equation (8)

$$H(\psi) = \frac{1}{\sin\frac{\psi}{2}} - \ln\left(1 + \frac{1}{\sin\frac{\psi}{2}}\right) \tag{8}$$

Restoration of Global Geopotential Model effect on local geoid (N_1) to obtain definitive undulation (N) can be calculated using equation (9)

$$N = N_{res}(\varphi, \lambda) + N_1 \tag{9}$$

where

$$N_1(\varphi, \lambda) = R.\sum_{n=2}^{N} \sum_{m=0}^{l} \bar{P}_{nm}.(\sin\varphi).(\Delta \bar{C}_{nm}.\cos m\lambda + \Delta \bar{S}_{nm}.\sin m\lambda) \tag{10}$$

3. Method of research

Geoid precision analysis occupies comparison of gravimetric and geometric geoid models. Gravimetric geoid modeling requires terrestrial gravity data, global geopotential model, and geodetic coordinates of gravity point. Geometric geoid is obtained by calculating the difference between geodetic height and orthometric height. Geodetic height was measured using relative static method of GNSS, while orthometric height was measured by Wild NAK2 spirit level.

Global geopotential model used in this research is data GECO created from the combination of data EGM2008 with data from the GOCE satellite. GECO has a harmonic coefficients to degree 2190 representing an area of 5 arc minute spatial resolution. Gravity data measured using Gravimeter Scintrex CG-5 with accuracy of 5 microgal. First geoid was calculated using gravity data measured at 62 locations with a density of about 2 km, as shown in Figure 1. Second geoid was calculated using gravity data measured at 186 points which was concentrated in the city of Semarang with additional measurements at the points until it reached an area of 1 arc degree squares, as shown in Figure 2.

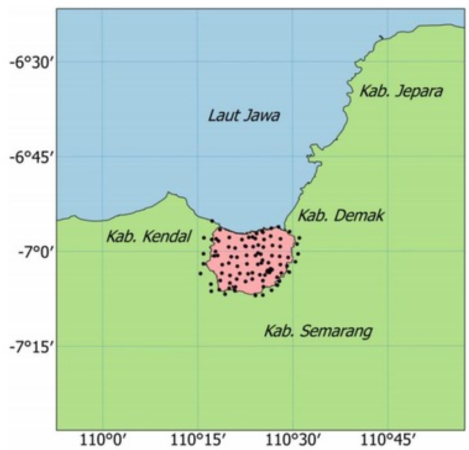

Figure 1. Gravity measurement location for 15 arc minute geoid modelling

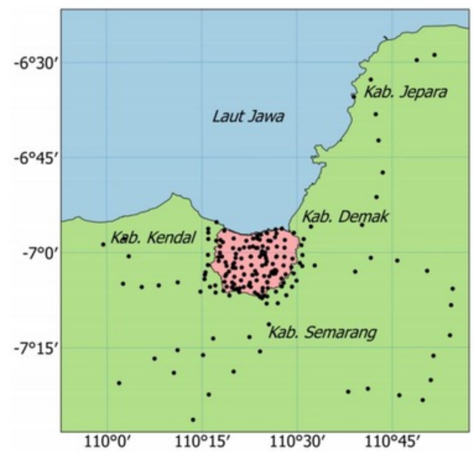

Figure 2. Gravity measurement location for 1 arc degree geoid modelling

Geodetic coordinates of gravity measurement locations was measured accurately using Topcon Hiper II GNSS receivers that recorded the GPS and GLONASS satellite signals. Measurements were made with rapid static method that was connected to GNSS CORS station. Reduction baseline performed using software Topcon Tools 8. Gravsoft 2.9.7 was occupied for gravimetric geoid calculation, especially GEOEGM and STOKES modules. GEOEGM was used to calculate the gravity disturbance and gravimetric geoid undulation of global geopotential model. For this research purpose, stokes function in STOKES module was modified to Hotine function.

4. Results

According to GNSS measurement, geodetic elevations of gravimetric stations were between 26.741 m to 356.04 m above the WGS84 reference ellipsoid. To obtain high accuracy positioning, the satellite observations carried out for 1 to 4 hours to ensure the value of the phase integer ambiguity was resolved. Solving phase ambiguity in Topcon Tools 8.5 produced geodetic coordinates with horizontal accuracy from ±0.004 m to ±0.087 m, while vertical accuracy from ±0.006 m to ±0.058 m.

Measurement of sprit level was divided into three loops of measurement, namely: northern loop, southern loop, and UNDIP loop. Path length of sprit level in the northern loop was 13.805 km with disclosure about +0.004 m. Based on SNI 19-6988-2004, the leveling net was fit the LAA classification. Length of southern loop is 51.520 km with misclosure +0.096 m which fit to LD classification.

Range of geoid undulation based on GECO data only was from 24.6 m to 25.4 m, as shown in Figure 3. Geoid undulation rose dramatically towards the south or towards Mount Ungaran. Drastic increase in geoid undulations was found in the northeast or the direction of Mount Muria in Jepara. Visually, the geoid undulation in Semarang and surrounding areas were modeled from GECO had a similar shape to the geoid obtained from direct measurements by Scintrex CG-5, as shown in Figure 4.

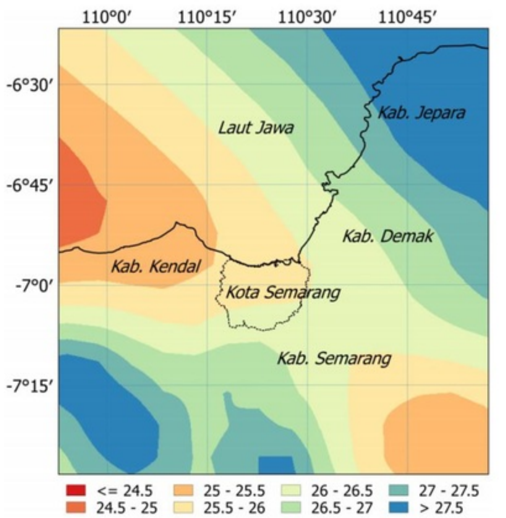

Figure 3. Geoid undulation from GECO only

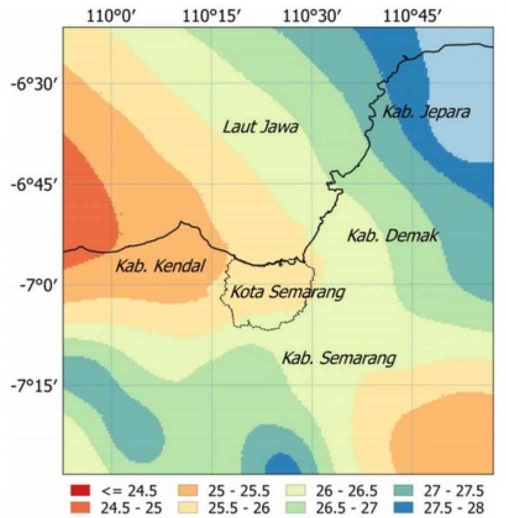

Figure 4. Geoid undulation from combination of GECO and terrestrial gravity data

Accuracy of gravimetric geoid was validated using 30 points of geometric geoid which was measured using GNSS and spirit leveling. Validation point was distributed along southern loop. Accuracy of geoid modeled from combination of GECO and 62 gravity data was ± 0.045 m, while geoid modeled using 186 gravity data was ± 0.044 m.

5. Conclusion

The conclusions of this research are:

• Accuracy of modeling the geoid with Hotine approach is suitable for engineering purposes
• Accuracy of geoid can be improved by enlarging gravity data measurement.

Acknowledgement

Thanks to Prof. Rene Forsberg of DTU who has allowed the use of Software Gravsoft. Thanks to Badan Informasi Geospasial (Geospatial Information Agency of Indonesia) for providing the gravimeter.

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